Solution by S. A. COREY, Hiteman, Iowa.

$$\int_0^{\frac{1}{2}\pi} d\theta\sqrt{[1+\sin^2\theta(1-4\cos\theta)]} \equiv \int_0^{\frac{1}{2}\pi} d\theta\sqrt{(1\tfrac{1}{2}-\cos\theta-\tfrac{1}{2}\cos2\theta+\cos3\theta)} \;\ldots\ldots\ldots\ldots(1).$$

The value of the second member of (1) is easily computed by aid of the formula

$$f(x)-f(0)=\frac{x}{m.2}\left\{[f'(x)+f'(0)]+2\left[f\left(\frac{x}{m}\right)+f'\left(\frac{2x}{m}\right)+f'\left(\frac{3x}{m}\right)+\ldots\ldots\right.\right.$$

$$\left.\left.\ldots\ldots+f'\left(\frac{m-1}{m}x\right)\right]\right\}-\frac{B_1x^2}{m^2.2!}[f''(x)-f''(0)]+\frac{B_2x^4}{m^4.4!}[f^{\text{iv}}(x)-f^{\text{iv}}(0)]+\ldots\ldots$$

$$\ldots\ldots+(-1)^n\frac{B^nx^{2n}}{m^{2n}.(2n)!}[f^{2n}(x)-f^{2n}(0)]+\ldots\ldots$$

B_1, B_2, ..., being Bernoulli's numbers.

Taking $m=5$, and $n=2$, the value of the definite integral is found to be 1.17066. For greater accuracy, larger values of m and n may be taken.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let $\sin^2\theta(1-4\cos\theta)=x$. Then by expanding the expression we get

$$(1+x)^{\frac{1}{2}}=1+\tfrac{1}{2}x-\tfrac{1}{8}x^2+\tfrac{1}{16}x^3-\tfrac{5}{128}x^4+\tfrac{7}{256}x^5-\ldots\ldots$$

$$\int_0^{\frac{1}{2}\pi}d\theta=\tfrac{1}{2}\pi=1.5708;\qquad \tfrac{1}{2}\int_0^{\frac{1}{2}\pi}xd\theta=\tfrac{1}{8}\pi-\tfrac{2}{3}=-.2740;$$

$$-\tfrac{1}{8}\int_0^{\frac{1}{2}\pi}x^2d\theta=-\frac{11\pi}{128}=-.0699;\qquad \tfrac{1}{16}\int_0^{\frac{1}{2}\pi}x^3d\theta=\frac{35\pi}{512}-\tfrac{59}{252}=-.0194;$$

$$-\tfrac{5}{128}\int_0^{\frac{1}{2}\pi}x^4d\theta=\tfrac{215}{792}-\frac{2975\pi}{32768}=-.0138;$$

$$\tfrac{7}{256}\int_0^{\frac{1}{2}\pi}x^5d\theta=\frac{16401\pi}{131072}-\frac{54761}{137280}=-.0059;$$

$$\therefore\int_0^{\frac{1}{2}\pi}\sqrt{[1+\sin^2\theta(1-4\cos\theta)]}d\theta=1.1858-.$$

Also solved by the Proposer.

MECHANICS.

170. Proposed by ELISHA S. LOOMIS, Ph. D., Berea, Ohio.

Two angles of iron, A_1CD and A_1CA_3, move freely on a pivot at C. Rods B_1A_1 and B_1A_3 are attached respectively at A_1 and at some point A_3 so that

when B_1 moves along the rod CR, which is perpendicular to A_1A_4, CD and CA_3 shall coincide in position with CE which is perpendicular to rod KR. When angle A_1CD is 135° find CA_3 in terms of CA_1.

Also find the following:

a). That value of CB_1 which will require least effort exerted at B_1 to cause CA_3 to take the position CA_4.

b). That value of CB_1 which will cause B_2A_2, if produced, to pass through the point A_1.

c). As CB_1 varies in value, what is the locus of the intersection of A_1B_1 and A_2B_2? Of B_1A_3 and B_2A_4?

d). Suppose angle A_1CD to be any other angle than 135°, then find CA_3 in terms of CA_1.

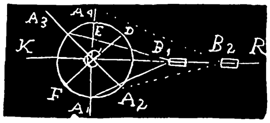
Solution by the PROPOSER.*

Determination of the value of CA_3 in terms of CA_1 when angle $A_1CD=135°$.

a). In triangle CA_1B_1, let $CA_1=r$ and $CB_1=vr$. Then $A_1B_1=r\sqrt{(1+v^2)}$.

b). In triangle CA_2B_2, $CA_2=r$, $A_2B_2=r\sqrt{(1+v^2)}$, and angle $A_2CB_2=45°$. Then, by trigonometry, $\sin B_2=\dfrac{r\sin C}{r\sqrt{(1+v^2)}}=\dfrac{1}{\sqrt{2}\sqrt{(1+v^2)}}$.

$$\therefore CB_2A_2=\sin^{-1}\frac{1}{\sqrt{2}\sqrt{(1+v^2)}}.$$



$$\therefore CA_2B_2=135°-\sin^{-1}\frac{1}{\sqrt{2}\sqrt{(1+v^2)}}.$$

$$\therefore CB_2=\frac{r\sqrt{(1+v^2)}\sin\left(137°-\sin^{-1}\dfrac{1}{\sqrt{2}\sqrt{(1+v^2)}}\right)}{\sin 45°}$$

$$=r\sqrt{2}\sqrt{(1+v^2)}\sin\left(135°-\sin^{-1}\frac{1}{\sqrt{2}\sqrt{(1+v^2)}}\right)$$

$$=kr, \text{ in which } k=\sqrt{2}\sqrt{(1+v^2)}\sin\left(135°-\sin^{-1}\frac{1}{\sqrt{2}\sqrt{(1+v^2)}}\right).$$

c). In triangle CB_1A_3, $CB_1=vr$ and angle $B_1CA_3=135°$. Draw A_3H perpendicular to CK. Then $CA=A_3H$. Let $CA_3=xr$, and $CH=p=xr/\sqrt{2}$.

$$\therefore (B_1A_3)^2=(CB_1)^2+(CA_3)^2+2CB_1\times CH=(vr)^2+(xr)^2+\sqrt{2}\,xvr^2$$
$$=r^2(v^2+x^2+\sqrt{2}\,vx).$$

d). In triangle B_2CA_4, $CA_4=CA_3=xr$, $B_1A_3=B_2A_4$, $CB_2=kr$, and angle $B_2CA_4=90°$. $(B_2A_4)^2=(B_2C)^2+(CA_4)^2=r^2(k^2+x^2)$.

$$\therefore r^2(v^2+x^2+\sqrt{2}\,vx)=r^2(k^2+x^2).\quad \therefore v^2+\sqrt{2}\,vx=k^2.\quad \therefore \sqrt{2}\,vx=k^2-v^2.$$

$$\therefore x=\frac{k^2-v^2}{\sqrt{2}\,v}.\quad \therefore xr=\frac{r\{[\sqrt{2}\sqrt{(1+v^2)}\sin(135°-\sin^{-1}\frac{1}{\sqrt{2}\sqrt{(1+v^2)}})]^2-v^2\}}{\sqrt{2}\,v}.$$

Also solved by G. B. M. Zerr.

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*Dr. Loomis writes us that the determination of CA_3 in terms of CA_1 is needed in a certain mechanism for producing dissolving views in a magic lantern. ED. E.